Interference Alignment for Two-User Two-Hop Interference X-Channel with Delayed and No CSIT

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***Abstract*—Capacity approximation of multi-hop unicast and multi-hop multicast channels is one of unsolved problems in information theory. Recently some researches investigate Degrees of Freedom (DoF) characterization and Interference Alignment schemes for these channels. However most of them assumed perfect instantaneous Channel State Information (CSI) at the relays and the transmitters. Due to practical limitations, like the delay and the rate limitation in the feedback links and the fading channels, it is difficult to provide perfect instantaneous CSI at the transmitters and even the relays. Achievable DoF by the IA schemes collapses greatly with imperfect CSI. It has been shown that the delayed CSI at the Transmitter (CSIT) can help to achieve higher DoF. In this paper, we investigate the DoF in the *two-user two-hop Interference X-channel* $(2\times2\times2-X)$ under two regimes of CSI availability: *delayed CSIT* and *no CSIT*. We present IA scenarios for each of these regimes which achieve the optimal Dof of $\frac{4}{3}$. In no CSIT case, we apply suitable strategy at the relays, which have delayed CSI, to compensate the effect of CSI absence in the transmitters from DoF point of view.**

***Index Terms*—Interference Alignment; No CSIT; Delayed CSIT; Two-User Two-Hop Relay X Channel; Optimal DoF.**

I. Introduction

Rapid progress in applications of wireless networks makes the interference phenomenon more and more challenging issue in proper operation of these networks. Interference is a inherit feature of wireless networks because of broadcast nature of nodes wireless communication on a common medium. The growing demand of large and crowd wireless networks should make concurrent communication of nodes possible, then these networks are interference-limited instead of noise-limited. In order to get more insight to design and analyze of such networks, capacity characterization of interference-limited networks is one of the most important problems in information theory domain. Interference channel (IC) is the best basic setup to model these networks.

Recently, the authors in [1] made great progress in approximating the capacity of two-user Gaussian channel within a constant gap. Also further progress in capacity characterization of some networks could be found in [2]–[4]. In addition to constant gap capacity approximation in [1], [5], Degrees of Freedom (DoF) is a powerful tool to approximate capacity of interference-limited wireless networks [6], [7]. DoF also known as multiplexing gain, number of resolvable signal dimensions, and capacity pre-log factor. Because a network has d degrees of freedom if and only if sum-capacity can be written in the form of the following equation.

$$C_{sum} = d.log(SNR) + o(log(SNR))$$

In [6], it has been mentioned that DoF provides clear insight to capacity approximation and asymptotic rate behavior in high SNR. In [8], [9], it has been shown that IA is a technique to achieve optimal DoF for some ICs. Unfortunately, in spite of recent progresses in capacity approximation of single-hop networks, capacity approximation of multi-hop multi-cast and uni-cast networks is an unsolved problem. First investigates on the multi-hop networks pioneered in [10]–[12] and great progresses pursued by Guo et. al. for DoF characterization of two-user two-hop wireless networks denoted as $2\times2\times2$, in [13], and then for multi-hop two-flow networks in [14]. These results were derived by assuming perfect instantaneous and even global CSI in applying the IA schemes. However in most of practical cases it is difficult to provide instantaneous or perfect CSI at the transmitters or the relays. At the presence of delayed CSI or no CSIT, achievable DoFs of these schemes considerably decrease. Thus, it is important to present schemes with ability to handle the interference efficiently in practical cases, like fading channel with small coherence time or delayed CSI feedback. Madah-Ali and Tse investigate the the delayed CSIT for Multiple-Iput Single-Output (MISO) Broadcast Channel (BC) and they show that the delayed CSIT is useful to achieve higher DoF for MISO BC [15]. Following the previous works, Vaze and Varanasi obtain DoF region of two-user Multiple-Input Multiple-Output(MIMO) BC under delayed CSIT situation in [16].

Unfortunately, in some practical cases, even the delayed CSIT is not available to transmitters. In these situations, by applying same IA scheme proposed for available delayed CSIT, DoF reduces to 1 and performance of network drastically degrades. To manage interference in these cases, Blind Interference Alignment (BIA) emerged. Wang et. al introduce the idea of BIA by staggered antennas in [17]. Then, Tian and Yener present a BIA method for K-user X-channel in [18]. They showed that in no CSIT case, by using half-duplex relays which have global CSI, one can achieve the same DoF as available delayed CSIT case. This work is extended in [19] to $M\times N$ X-channel aided by J multi-antenna half-duplex relays.

In this paper, motivated by the above works, we investigate the *two-user two-hop Interference X-channel* $(2\times2\times2-X)$

under delayed CSIT and no CSIT assumptions. DoF of two-hop two-user interference network under four CSI feedback scenarios named Shannon feedback, output feedback, delayed CSIT and limited Shannon feedback has been investigated in [20] and IA schemes for limited Shannon feedback and delayed CSIT have been presented. In this paper, we consider a $2\times2\times2-X$ channel to further study the optimal achievable DoF in two cases: (i) the delayed version of CSI is available at the transmitters, (ii) the transmitters have no side information however the relays have access to the delayed global CSI. In the first case, each transmitter know CSI of both hop with a finite delay. Moreover, they know received signals by one of relays whit a finite delay. The relays do not know CSI. The relays are full-duplex, however they are not instantaneous. In our second case, one of relays know CSI and received signal by other relay with a finite delay. In this case, the transmitters have no CSI and side information about received signal by the relays and receivers.

Comparing our CSIT availability cases with four CSI feedback scenarios in [20], we deduce that our delayed CSIT case is a weaker regime than the Shannon feedback. Moreover, our no CSIT case is a weaker regime than the limited Shannon feedback. In a weaker regime, under the same conditions, some of the nodes have a subset of CSIs compared to original channel . For example, in the Shannon feedback scenario in [20], the transmitters know the global CSI of both hops with finite delays, as well as what relays and receivers receive with finite delays. In addition, the relays know the second hop's delayed CSI. Whereas in the delayed CSIT case of our work, the transmitters only know the delayed CSI and relays' received signal with some finite delays; and the relays have no CSI. We show that the optimal achievable sum-DoF of $2\times2\times2$ in both cases of no CSIT and delayed CSIT is $\frac{4}{3}$. We present the IA scenarios for each case and show that these scenarios achieve optimal sum-DoF.

Rest of the paper is organized as follows. In Section II, we describe the system model and CSIT availability cases. In Section III, we state our main results for each case and in section IV, we present IA methods for both cases which achieve optimal sum-DoF. In Section V, we conclude the paper.

II. Network Model and Preliminaries

In this section, we describe our network model in details and explain the CSI availability cases. As we see in Fig. 1, the $2\times2\times2-X$ consists of two transmitters denoted as S_1 and S_2. Each transmitter wishes to transmit a message to each receiver denoted as D_1 and D_2. Thus, transmitters want to communicate with both receivers to transfer their corresponding messages, i.e., S_j wants to send m_{ij} to D_i for $i,j=1,2$. Since there is no direct link between the transmitters and the receivers, they communicate via intermediate nodes (relays) denoted as R_1 and R_2. Fig. 1 depict network in t^{th} channel use: the i^{th} transmitter (S_i) generates $X_{S_i}(t)\in\mathbb{C}$ based on its message and CSIT; then, it is sent towards the relays on the first hop. The channel coefficient between S_i and R_j at t^{th} channel use is denoted as $h_{ji}(t)\in\mathbb{C}$. A set of channel

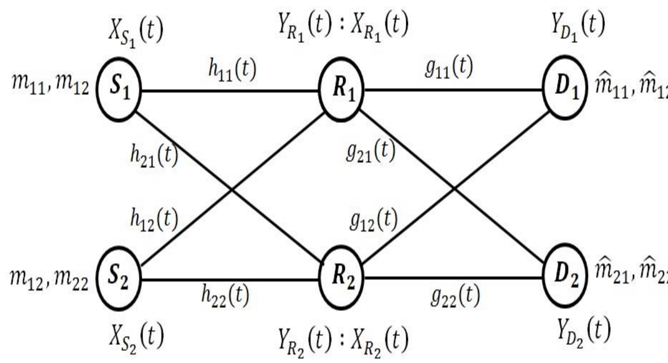

Fig. 1. Two-user two-hop interference X channel ($2\times2\times2-X$).

coefficients in $\{h_{ij}(t)\}_{i,j}$ shows the first hop. The i^{th} relay observe $Y_{R_i}(t)\in\mathbb{C}$ at the end of t^{th} channel use.

$$Y_{R_i}(t)=h_{i1}(t)X_{S_1}(t)+h_{i2}(t)X_{S_2}(t)+Z_{R_i}(t),i=1,2. \quad (1)$$

where $Z_{R_i}(t)\in\mathbb{C}$ is an independent and identically distributed (i.i.d) zero-mean Gaussian noise component. We consider full-duplex relays and the outputs of the relays depend on the received signals in the past channel uses. Then at t^{th} channel use, R_i based on its CSI and past received signals $\{Y_{R_i}(t-1),Y_{R_i}(t-2),...\}$ generates $X_{R_i}(t)\in\mathbb{C}$ and sends it over the second hop toward the receivers. The channel coefficient between R_i and D_j at t^{th} channel use is denoted by $g_{ji}(t)\in\mathbb{C}$. A set of channel coefficients in $\{g_{ij}(t)\}_{i,j}$ denotes the second hop. The D_i at t^{th} channel use receives $Y_{D_i}(t)$ and based on its CSI tries to extract its desire message(s).

$$Y_{D_i}(t)=g_{i1}(t)X_{R_1}(t)+g_{i2}(t)X_{R_2}(t)+Z_{D_i}(t),i=1,2. \quad (2)$$

where $Z_{D_i}(t)\in\mathbb{C}$ is an independent and identically distributed (i.i.d) zero-mean Gaussian noise component. All channel coefficients, $\{h_{ij}(t)\}_{i,j}$ and $\{g_{ij}(t)\}_{i,j}$, and additive white noises $Z_{R_i}(t)$ and $Z_{D_i}(t)$ are scalars with complex normal distributions with zero mean and unit variance $\mathcal{CN}(0,1)$ and they are i.i.d over i, j and t. We assume all transmitters, relays and receivers know these distributions. Moreover, there is an average power constraint P on $X_{R_i}(t)$ and $X_{S_i}(t)$.

$$\mathbb{E}|X_{S_i}(t)|,\mathbb{E}|X_{R_i}(t)|\le P;\forall i,t. \quad (3)$$

The sum-capacity C_{sum} is the maximum achievable sum-rate under average power constraint P. The Degree of Freedom (DoF) of intended network is defined as follows:

$$d=\lim_{P\to+\infty}\frac{C_{sum}(P)}{\log_2 P} \quad (4)$$

In addition to power constraint, sum-capacity depends on the structure of the network and CSI availability in nodes. Availability of CSI can help to achieve higher successful communication rate. In this paper, we investigate two regimes of CSI availability which are described in the rest of this section.

A. Delayed CSIT

In delayed CSI case, each transmitter knows both hop's with a finite delay. Also S_i knows the received signal by R_i's received signal for $i = 1, 2$ with a finite delay. However relays have no information about the channels and the received signals at the receivers. Without loss of generality, we assume finite delay is equal to the one channel use in the rest of the paper. In other words, at t^{th} channel use, the S_i knows $\{h_{ij}(t-1)\}_{i,j}$,$\{g_{ij}(t-1)\}_{i,j}$ and $Y_{R_i}(t-1)$ for $i, j = 1, 2$.

B. No CSIT

In no CSIT case, the transmitters have no information about channels of first and second hops. They also have no side information about what are received at the relays and the receivers. In this case, the relays play the main role instead of the transmitters. We investigate a special case where one of the relays is more powerful and has side information and the other one does not know CSI and the received signal at receivers. The powerful relay knows CSI of both hops with a finite delay as well as what the other relay receives (with a finite delay). Same as previous case, we assume unit finite delay. With this assumption, one of the relays (i.e., R_1) knows $\{h_{ij}(t-1)\}_{i,j}$,$\{g_{ij}(t-1)\}_{i,j}$ and $Y_{R_2}(t-1)$ at t^{th} channel use for $i, j = 1, 2$.

As we described in this section, at the delayed CSIT case, the transmitters which have delayed CSI and side information about received signal by one of relays, must produce proper signals to make enable the receivers to recover their desired messages. It is obvious that without CSI at transmitters in this case, DoF reduces. In the second case, we consider this situation. We investigate that can we compensate DoF reduction due to lack of CSIT by equipping relay(s) with some capabilities? Then in second case, we consider one of the relays has delayed CSI and side information about received signal by other relay, however, other relay do know know CSI locally or globally. This relay just use AF scenario. In this situation, relay which know CSI, has main role in IA. Since in this case, CSIT is not available, our IA method for this case is a kind of BIA. In section IV, we show that suitable strategy at powerful relay, make enable the receivers to recover their desired message. Also, by our strategies in cooperation of different nodes, our IA scheme achieves the optimal DoF of previous case (i.e.,delayed CSIT case). Since there is no time limit on decoding, we can assume available CSI at the receivers is instantaneous. In other words, receivers can wait until receiving required CSI before starting to decode. Hence, without loss of generality, we assume that the receivers have global CSI instantaneously. In the next section, we state our main result in both cases.

III. Main Results

Theorem 1: The optimal sum-DoF of two-user two-hop interference X-channel in both cases of no CSIT and delayed CSIT is $\frac{4}{3}$.

Proof. If we compare two described cases with four feedback scenarios in [20], it can be understood that our case of delayed CSIT is a weaker regime than the Shannon feedback in [20]. Because, in the Shannon feedback case, the relays know CSI of second hop and what destinations receive with a finite delay; however, in the delayed CSIT case the relays have no information about the channel and what received at the receivers. Moreover in our case, each transmitter knows the received signal at one of the relays instead of both relays. On other hands, with a simply comparing the no CSIT and limited Shannon feedback cases, we can deduce that no CSIT is a weaker regime. Because, in the limited Shannon feedback case of [20],in addition to all side information of our powerful relay (in no CSIT case), the powerful relay knows the received signals at receivers with a finite delay. Therefore, the following inequality between DoF of our cases and the Shannon and limited Shannon feedback cases (in X-mode using of network) can be deduced:

$$d^{X-NCSIT} \leq d^{X-LSF}; \tag{5}$$

$$d^{X-DCSIT} \leq d^{X-SF}; \tag{6}$$

where $d^{X-NCSIT}$ and $d^{X-NCSIT}$ denote the optimal DoF of no CSIT and delayed CSIT in $2 \times 2 \times 2 - X$, respectively. Moreover, d^{X-LSF} and d^{X-SF} denote the optimal DoF of $2 \times 2 \times 2$ in X-mode with limited Shannon feedback and Shannon feedback, respectively.

In [20], the authors noted that maximum achievable DoF of the $2 \times 2 \times 2$ channel with limited Shannon feedback and Shannon feedback in X-mode is $\frac{4}{3}$. However, they did not present any achievable scheme for Shannon feedback and limited Shannon feedback cases in X-mode. Based on this result we can present an upper bound on the maximum achievable DoF for Delayed and no CSIT cases;as:

$$d^{X-NCSIT} \leq \frac{4}{3}; \tag{7}$$

$$d^{X-DCSIT} \leq \frac{4}{3}; \tag{8}$$

In the next section, we present two IA schemes (one for each case) that achieve $\frac{4}{3}$ DoF in both cases. This completes the proof.■

IV. Interference Alignment Schemes

In this section, we present two IA schemes for no CSIT and delayed CSIT cases. We show that our presented schemes achieve $\frac{2}{3}$ DoF for each receiver and $\frac{4}{3}$ totally. Since this is the upper bound on the DoF for both cases, the optimal DoF of this network in no CSIT and delayed CSIT regimes is $\frac{4}{3}$ and this optimal DoF can be achieved by these two presented IA schemes.

A. IA for Delayed CSIT

Since our goal is to achieve $\frac{4}{3}$ DoF, we must successfully transmit two messages to each receiver in three time slots. Each transmitter has a unique message for each receiver. m_{ji} is the message of S_i intended for D_j. Hence, the desired

TABLE I
SCHEDULING OF IA FOR DELAYED CSIT

Phase	Phase#1	Phase#2	Phase#3	Phase#4
Time slots	$t_1 = 1, 2$	$t_2 = 1$	$t_1 = 3$	$t_2 = 2, 3$

messages for D_1 are m_{11} from S_1 and m_{12} from S_2. Also, desired messages for D_2 are m_{21} from $S_1$and m_{22} from S_2. Our proposed IA scheme consists of three time slots in each hop. We denote the the index of channel use in first hop by t_1 and in the second hop by t_2. In our IA scheme, first $t_1 = 1, 2$ occurs and next $t_2 = 1$, $t_1 = 3$ and $t_2 = 2, 3$ occur. According to this scheduling, we can divide our alignment scheme into four phases. Scheduling of our IA scheme is shown in table I. Note that, in transmitting the b^{th} message, the phases 2 and 3 occur at the same time. However, while Phase 4 for the b^{th} message is in progress in the second hop, the $b+1^{th}$ message is transmitting in the first hop. In fact, the relays transmit with two time slots delay. We remark that the relays work in full-duplex mode and they use Amplify-Forward (AF) strategy for relaying. Now we explain the details of each phase.

Phase#1- This phase consists of two time slots or (i.e., channel uses) $t_1 = 1, 2$ in the first hop. In the first time slot, the transmitters send the sum of their messages and in the second time slot, they send differences of their messages toward the relays over the first hop. Hence, in $t_1 = 1$, S_1 sends $m_{11} + m_{21}$ and S_2 sends $m_{12} + m_{22}$. In the next time slot, i.e., $t_1 = 2$, S_1 sends $m_{11} - m_{21}$ and S_2 sends $m_{12} - m_{22}$. We omit additive white Gaussian noise in following equations, since we investigate DoF of the network in asymptotic high SNR. Then at the end of first time slot, the relays receive following signals:

$$Y_{R_1}(1) = h_{11}(1)(m_{11} + m_{21}) + h_{12}(1)(m_{12} + m_{22}). \quad (9)$$

$$Y_{R_2}(1) = h_{21}(1)(m_{11} + m_{21}) + h_{22}(1)(m_{12} + m_{22}). \quad (10)$$

In the second time slot, R_1 and R_2 receive $Y_{R_1}(2)$ and $Y_{R_2}(2)$, respectively.

$$Y_{R_1}(2) = h_{11}(2)(m_{11} - m_{21}) + h_{12}(2)(m_{12} - m_{22}). \quad (11)$$

$$Y_{R_2}(2) = h_{21}(2)(m_{11} - m_{21}) + h_{22}(2)(m_{12} - m_{22}). \quad (12)$$

Phase#2- This phase consists of one time slot ($t_2 = 1$). In this phase, we aim to create a combination of four messages at the receivers. For this purpose, it is sufficient for the relays to send a linear combination of their received signals at phase 1. We assume that in this phase, each relay sends its received signal of the first time slot of hop one (i.e., $t_1 = 1$). Thus in $t_2 = 1$, R_1 sends $Y_{R_1}(1)$ as $X_{R_1}(1)$ and R_2 sends $Y_{R_2}(1)$ as $X_{R_2}(1)$. At the end of this phase D_1 and D_2 receive $Y_{D_1}(1)$ and $Y_{D_2}(1)$, respectively:

$$Y_{D_1}(1) = g_{11}(1)X_{R_1}(1) + g_{12}(1)X_{R_2}(1). \quad (13)$$

$$Y_{D_2}(1) = g_{21}(1)X_{R_1}(1) + g_{22}(1)X_{R_2}(1). \quad (14)$$

Substituting $X_{R_1}(1)$=$Y_{R_1}(1)$ and $X_{R_2}(1)$=$Y_{R_2}(1)$ results in:

$$\begin{aligned} Y_{D_1}(1) = \{&g_{11}(1)h_{11}(1) + g_{12}(1)h_{21}(1)\}m_{11}+ \\ &\{g_{11}(1)h_{11}(1) + g_{12}(1)h_{21}(1)\}m_{21}+ \\ &\{g_{11}(1)h_{12}(1) + g_{12}(1)h_{22}(1)\}m_{12}+ \\ &\{g_{11}(1)h_{12}(1) + g_{12}(1)h_{22}(1)\}m_{22} = \\ &C^1_{11}m_{11} + C^1_{21}m_{21} + C^1_{12}m_{12} + C^1_{22}m_{22} \quad (15) \end{aligned}$$

where for simplicity, we denote the coefficient of m_{ij} by C^1_{ij}. Similarly, for D_2 we have:

$$\begin{aligned} Y_{D_2}(1) = \{&g_{21}(1)h_{11}(1) + g_{22}(1)h_{21}(1)\}m_{11}+ \\ &\{g_{21}(1)h_{11}(1) + g_{22}(1)h_{21}(1)\}m_{21}+ \\ &\{g_{21}(1)h_{12}(1) + g_{22}(1)h_{22}(1)\}m_{12}+ \\ &\{g_{21}(1)h_{12}(1) + g_{22}(1)h_{22}(1)\}m_{22} = \\ &C^2_{11}m_{11} + C^2_{21}m_{21} + C^2_{12}m_{12} + C^2_{22}m_{22} \quad (16) \end{aligned}$$

We can rewrite (15) and (16) according to the desired and unintended messages for related receivers. For D_1, combination of m_{11} and m_{12} is desired and combination of m_{21} and m_{22} make interference and similarly for D_2, combination of m_{21} and m_{22} is desired and combination of m_{11} and m_{12} make interference.

$$Y_{D_1}(1) = C^1_{11}m_{11} + C^1_{12}m_{12} + C^1_{21}m_{21} + C^2_{22}m_{22} = U_1 + I_1 \quad (17)$$

$$Y_{D_2}(1) = C^2_{21}m_{21} + C^2_{22}m_{22} + C^2_{11}m_{11} + C^2_{12}m_{12} = U_2 + I_2 \quad (18)$$

where I_i and U_i show the interference and the desired signals for D_i, respectively. The goal of two remaining phases is sending these interference parts of the received signals. If a receivers like D_1 knows both I_1 and I_2, it can extract U_1. Note that D_1 knows channel coefficients of the previous time slots. So, it can extract its desired messages from the set of two equations with two unknowns as desired messages.

Phase#3-This phase consists of one time slot in first hop (i.e., $t_1 = 1$). In this phase the transmitters send the interference part of the receiver's signals in phase 2. For this purpose, S_1 generates I_1 and S_2 generates I_2. To make this generation possible, each transmitter has to know one of the other transmitter's messages. For example, D_1 must know m_{22} to make I_1.

As explained in Section II in the delayed CSIT case, transmitters know CSI of phases. In addition, they know $Y_{R_i}(t)$ for $i, t = 1, 2$. Using these information, the transmitters can generate the interference parts. We describe this procedure for S_1. S_1 can calculate $m_{12}+m_{22}$ as well as $m_{12}-m_{22}$ from (9) and(11),respectively. Then, it can easily extract both messages of other source m_{12} and m_{22}. Using a similar procedure, S_2 obtains m_{11} and m_{21}. Then S_1 and S_2 make I_1 and I_2, respectively, according to (19) and (20).

$$\begin{aligned} I_1 = \{&g_{11}(1)h_{11}(1) + g_{12}(1)h_{21}(1)\}m_{21}+ \\ &\{g_{11}(1)h_{12}(1) + g_{12}(1)h_{22}(1)\}m_{22} \quad (19) \end{aligned}$$

$$I_2 = \{g_{21}(1)h_{11}(1) + g_{22}(1)h_{21}(1)\}m_{11} + \{g_{21}(1)h_{12}(1) + g_{22}(1)h_{22}(1)\}m_{12} \quad (20)$$

After calculating interference parts, S_1 sends I_1 and S_2 sends I_2 to relays. At the end of this phase, relays receive the following signals:

$$Y_{R_1}(3) = h_{11}(3)I_1 + h_{12}(3)I_2 \quad (21)$$

$$Y_{R_2}(3) = h_{21}(3)I_1 + h_{22}(3)I_2 \quad (22)$$

Phase#4- This phase consists of two time slots(i.e., $t_2 = 2, 3$). In $t_2 = 2$, R_2 remains silent and R_1 sends $Y_{R_1}(3)$. Similarly, at $t_2 = 3$, R_1 remains silent and R_2 sends $Y_{R_2}(3)$. Then the receivers receive proper signals to recover interference parts I_1 and I_2. At the end of $t_2 = 2$, D_1 and D_2 receive $Y_{D_1}(2)$ and $Y_{D_2}(2)$, respectively:

$$Y_{D_1}(2) = g_{11}(2)h_{11}(3)I_1 + g_{11}(2)h_{12}(3)I_2; \quad (23)$$

$$Y_{D_2}(2) = g_{21}(2)h_{11}(3)I_1 + g_{21}(2)h_{12}(3)I_2; \quad (24)$$

At the end of $t_2 = 3$, the receivers receive signals according to follow:

$$Y_{D_1}(3) = g_{12}(3)h_{21}(3)I_1 + g_{12}(3)h_{22}(3)I_2; \quad (25)$$

$$Y_{D_2}(3) = g_{22}(3)h_{21}(3)I_1 + g_{22}(3)h_{22}(3)I_2; \quad (26)$$

Since the receivers know global CSI (instantaneously), D_1 with help of (23) and (25) can calculate both I_1 and I_2. In a similar manner, D_2 extracts I_2 and I_1 from (24) and (26). Then the receivers start to recover their desired messages. Here, We explain the procedure for D_1 in details. The case of D_2 similarly follows.

Knowing I_1, D_1 first calculates $U_1 = Y_{D_1}(1) - I_1$ by subtracting its interference part from its received signal in the first time slot of hop two. U_1 is a combination of desired messages. On the other hand, the interfering messages for the other receiver are desired messages for this receiver. In fact, I_2 is another linear combination of desired messages of D_1. Thus D_1 solves the following system of two equations with two unknown to recover its desired messages:

$$Y_{D_1}(1) - I_1 = \{g_{11}(1)h_{11}(1) + g_{12}(1)h_{21}(1)\}m_{11} + \{g_{11}(1)h_{12}(1) + g_{12}(1)h_{22}(1)\}m_{12}; \quad (27)$$

$$I_2 = \{g_{21}(1)h_{11}(1) + g_{22}(1)h_{21}(1)\}m_{11} + \{g_{21}(1)h_{12}(1) + g_{22}(1)h_{22}(1)\}m_{12}; \quad (28)$$

Using a similar procedure, D_2 solves the following system of two equations to recover its desired messages:

$$Y_{D_2}(1) - I_2 = \{g_{21}(1)h_{11}(1) + g_{22}(1)h_{21}(1)\}m_{21} + \{g_{21}(1)h_{12}(1) + g_{22}(1)h_{22}(1)\}m_{22} \quad (29)$$

$$I_1 = \{g_{11}(1)h_{11}(1) + g_{12}(1)h_{21}(1)\}m_{21} + \{g_{11}(1)h_{12}(1) + g_{12}(1)h_{22}(1)\}m_{22} \quad (30)$$

TABLE II
TRANSMITTED SIGNALS BY TRANSMITTERS IN THE FIRST HOP

Time Slot	$X_{S_1}(t_1)$	$X_{S_2}(t_1)$
$t_1 = 1$	$m_{11} + m_{21}$	$m_{12} + m_{22}$
$t_1 = 2$	$m_{11} - m_{21}$	$m_{12} - m_{22}$
$t_1 = 3$	I_1	I_2

TABLE III
TRANSMITTED SIGNALS BY RELAYS IN THE SECOND HOP

Time Slot	$X_{R_1}(t_2)$	$X_{R_2}(t_2)$
$t_2 = 1$	$Y_{R_1}(1)$	$Y_{R_2}(1)$
$t_2 = 2$	$Y_{R_1}(3)$	$\varnothing$
$t_2 = 3$	$\varnothing$	$Y_{R_2}(3)$

Thus, each receiver recovers its two desired messages and each transmitter sends its messages to intended receiver successfully. We send two messages to each receiver over three time slots and we get $\frac{4}{3}$ DoF. As we said in Theorem 1, this is optimal DoF of $2 \times 2 \times 2 - X$ with delayed CSIT. Thus, our presented scheme is optimal from DoF point of view. We summarize transmitted proper signals for alignment in the first and second hops, respectively, in tables II and III. In these tables and the rest of paper, $\varnothing$ means that the related node remains silent at the current time slot.

B. IA for No CSIT

In many practical cases, it is not possible for the transmitters to obtain CSI. This case is more convenient in fading channels with small coherence time. Here, we present an IA scheme consisting of two phases. Since our goal is to achieve DoF equal to $\frac{4}{3}$, we must transmit one message from each transmitter to each receiver. We must send these four messages during three time slots successfully, then each receiver achieves $\frac{2}{3}$ DoF and achievable sum-DoF equals to $\frac{4}{3}$. Since the optimal DoF of our network under no CSIT regime has an upper bound equal to $\frac{4}{3}$, we achieve maximum sum-DoF for t$2 \times 2 \times 2 - X$ in no CSIT case. Our presented scheme consists of two phases. Phase#1 consists of three time slots in the first hop(i.e., $t_1 = 1, 2, 3$) and Phase#2 consists of three time slots in the second hop (i.e., $t_2 = 1, 2, 3$).

Phase#1- In this phase, the transmitters send proper combination of their messages to the relays on the first hop. transmitted signals by each transmitter in the first hop is shown in table IV, briefly. In first time slot of phase#1, S_1 sends its message for D_1 (m_{11}) and S_2 remains silent. In the next time

TABLE IV
SENT SIGNALS BY TRANSMITTERS IN FIRST HOP

Time Slot	X_{S_1}	X_{S_2}
$t_1 = 1$	m_{11}	$\varnothing$
$t_1 = 2$	$\varnothing$	m_{12}
$t_1 = 3$	m_{21}	m_{22}

TABLE V
TRANSMITTED SIGNALS BY RELAYS IN THE SECOND HOP

Time Slot	$X_{R_1}(t_2)$	$X_{R_2}(t_2)$
$t_2 = 1$	$Y_{R_1}(1) + Y_{R_1}(3)$	$Y_{R_2}(2) + Y_{R_2}(3)$
$t_2 = 2$	I_1	$\varnothing$
$t_2 = 3$	I_2	$\varnothing$

slot, S_2 sends its message for D_1 and S_1 remains silent. In the third time slot both transmitters send their messages intended for D_2. Since our goal is to analyze the DoF of network in asymptotic high SNR, we omit additive white Gaussian noise in equations. At the end of the first and second time slots, the relays receive the following signals:

$$Y_{R_1}(1) = h_{11}(1)m_{11} \tag{31}$$

$$Y_{R_2}(1) = h_{21}(1)m_{11} \tag{32}$$

$$Y_{R_1}(2) = h_{12}(2)m_{12} \tag{33}$$

$$Y_{R_2}(2) = h_{22}(2)m_{12} \tag{34}$$

Moreover, at the end of phase#1, the relays receive a combination of messages intended for the second receiver:

$$Y_{R_1}(3) = h_{11}(3)m_{21} + h_{12}(3)m_{22} \tag{35}$$

$$Y_{R_2}(3) = h_{21}(3)m_{21} + h_{22}(3)m_{22} \tag{36}$$

We pursue two main goals in this phase. First, the transmitters send proper signals to the relays in order to make relays able to create a linear combination of four messages in each receiver. Second goal is to enable the relays to identify and generate the interference parts of the mentioned combination according to received signals from transmitters and their side information in last time slot of second phase. Then, we choose the transmitting signals with the above features.

Phase#2- This phase consists of three time slots in the second hop (i.e., $t_2 = 1, 2, 3$). Similar to the previous IA scheme, here, first we send signals such that the receivers receive a linear combination of four messages. This combination includes two part, linear combination of desired message U_i and linear combination of unintended messages I_i, $i = 1, 2$. In the next two time slots, the relays' duty is to make the receivers to understand I_1 and I_2. After this process, the receivers can extract their desired messages from linear combination with help of CSI and knowing I_1 and I_2. In table V, we summarize the transmitted signals by the relays in different time slots on the second hop. As we mentioned before, at the end of first time slot (i.e., $t_1 = 1$) the receivers receive signals according to (37) and (38). These signals make a linear combination of four messages of transmitters.

$$\begin{aligned} Y_{D_1}(1) = \{g_{11}(1)h_{11}(1)\}m_{11} + \{g_{12}(1)h_{22}(2)\}m_{12} \\ + \{g_{11}(1)h_{11}(3) + g_{12}(1)h_{21}(3)\}m_{21} + \\ \{g_{11}(1)h_{12}(3) + g_{12}(1)h_{22}(3)\}m_{22} = \\ C^1_{11}.m_{11} + C^1_{12}.m_{12} + C^1_{21}.m_{21} + C^1_{22}.m_{22} = U_1 + I_1 \end{aligned} \tag{37}$$

$$\begin{aligned} Y_{D_2}(1) = \{g_{21}(1)h_{11}(1)\}m_{11} + \{g_{22}(1)h_{22}(2)\}m_{12} \\ + \{g_{21}(1)h_{11}(3) + g_{22}(1)h_{21}(3)\}m_{21} + \\ \{g_{21}(1)h_{12}(3) + g_{22}(1)h_{22}(3)\}m_{22} = \\ C^2_{11}.m_{11} + C^2_{12}.m_{12} + C^2_{21}.m_{21} + C^2_{22}.m_{22} = I_2 + U_2 \end{aligned} \tag{38}$$

where I_i and U_i are interference and desired parts of the received signal by D_i and C^i_{jk} is m_{jk} coefficient in D_i for $k, j, i = 1, 2$.

$$U_1 = C^1_{11}.m_{11} + C^1_{12}.m_{12}; I_1 = C^1_{21}.m_{21} + C^1_{22}.m_{22} \tag{39}$$

$$U_2 = C^2_{21}.m_{21} + C^2_{22}.m_{22}; I_2 = C^2_{11}.m_{11} + C^2_{12}.m_{12} \tag{40}$$

In the second time slot R_1 which has CSI and received signals of R_2 in past time slots, must send I_1. Hence, this relay must know m_{21} and m_{22} to create I_1. Since R_1 knows channel coefficient in phase#1 , it can easily calculate m_{11} and m_{12} from $Y_{R_1}(1)$ and $Y_{R_1}(2)$. In addition to its received signal in the third time slot of hop one, R_1 knows $Y_{R_2}(3)$. Then with these two signals, it can extract m_{21} and m_{22} by solving a simple system of two equations with two unknown. Thus after these efforts R_1 knows all messages of transmitter. Also at current time slot R_1 knows CSI of second hop in the first time slot. Therefor, it can calculate C^i_{jk} easily. In the second and third time slots, R_1 generates I_1 and I_2 by knowing messages and their coefficients in (39) and (40) and it sends interference parts to the receivers.

At the last time slot, the signals received at receivers contain interference parts. With knowing CSI of both hop, the receivers can recover both I_1 and I_2. The remaining procedure is same as the delayed CSIT case. Here, we only explain for D_1. The other one is similar.

At the end of two last time slots, the received signals at receivers are:

$$Y_{D_1}(2) = g_{11}(2).I_1 \tag{41}$$

$$Y_{D_2}(2) = g_{21}(2).I_1 \tag{42}$$

$$Y_{D_1}(3) = g_{12}(3).I_2 \tag{43}$$

$$Y_{D_2}(3) = g_{22}(3).I_2 \tag{44}$$

Since D_1 knows CSI of second hop, it can extract I_1 and I_2 from (41) and (43), respectively. After calculating interference parts, D_1 calculates it desired part of the received signal in (37).

$$U_1 = Y_{D_1}(1) - I_1 = C^1_{11}.m_{11} + C^1_{12}.m_{12} \tag{45}$$

In addition D_1 knows I_2 now.

$$I_2 = C^2_{11}.m_{11} + C^2_{12}.m_{12} \tag{46}$$

Since D_1 knows CSI of first time slot in second hop, it knows message coefficients in (45) and (46). Therefore this destination can recover its desired messages (i.e., m_{11} and m_{12}) by solving a simple system of two equations includes (45) and (46) with m_{11} and m_{12} as two unknowns. Then, D_1 recovers its two desire messages successfully in three time slots. With the presented scheme, each transmitter can send a message to each receiver. Therefore, our presented scheme achieved DoF equals to $\frac{2}{3}$ for each receiver and achieved sum-DoF equals to $\frac{4}{3}$. Since $\frac{4}{3}$ is also an upper bound for sum-DoF, the optimal sum-DoF of $2 \times 2 \times 2 - X$ under no CSIT regime is equal to $\frac{4}{3}$. Therefore, our presented scheme is optimal from DoF point of view. We remark that one can consider this state as general case of when both relays have CSI. Therefore, our presented scheme is applicable on in similar networks with relays which both of them know delayed CSI.

V. Conclusion

In this paper, we investigated the two-user two-hop interference X-channel. We considered two practical cases where the transmitters cannot access the instantaneous CSI. These two regimes were named delayed CSIT and no CSIT. In delayed CSIT case, transmitter have main role in IA and they use their side information to make enable the receivers to recover their intended messages. However, in the second case, CSI are not available for transmitters. In this situation without further actions DoF reduces to 1. In this situation, we mentioned sufficient side information for one of relays to compensate the destructive effect due to absence of CSIT on DoF. With suitable strategies and proper side information in one of the relays, we compensated DoF reduction completely relative to first case. It is shown that under both regimes the maximum achievable sum-DoF has an upper bound equals to $\frac{4}{3}$. Next, we proposed two IA methods that both achieved $\frac{2}{3}$ DoF for each receiver and sum-DoF of $\frac{4}{3}$. Therefor, our presented IA schemes are optimal under related regime.

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