Blind Interference Alignment for Three-User Multi-Hop SISO Interference Channel

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***Abstract*—Interference Alignment (IA) is a rather new technique to achieve higher Degrees-of-Freedom (DoF) compared to traditional orthogonal naive methods, like, Time Division Multiple Access (TDMA). In many IA schemes, Instantaneous Channel State Information at Transmitter (CSIT) is mandatory. However, this requirement becomes an obstacle to implement those methods in practice. In some practical scenarios, it is not possible to provide instantaneous or even delayed CSIT. Recently, Blind Interference Alignment (BIA) has been introduced as a solution to align interference without CSIT. In spit of major advances in single-hop Interference Channels (ICs), few researches focused on multi-hop ICs. In this paper, we investigate the three-user multi-hop Single-Input Single-Ouput (SISO) IC. We present a BIA method to achieve the optimal DoF of this network without CSIT. Moreover, the relays do not have CSI and they using Amplify-and-Forward (AF) relaying strategy. Our method is based on using multi-mode antennas at the receivers. We also investigate a more practical case, in which the relays of an intermediate layer are equipped with multi-mode antennas and know global CSI, while the receivers use conventional antennas. In both cases, our methods achieve maximum achievable sum-DoF, equal to $\frac{6}{5}$. Hence, our proposed methods are optimal in the sense of DoF.**

***Index Terms*—Blind interference alignment; No CSIT; Three-user multi-hop SISO interference channel; Multi-mode antennas.**

I. Introduction

Increasing strives for communication in higher rate made great strides toward declaring the limits of traditional strategies. One of the impressed domains in information theory is capacity characterization of networks. In order to gain more insight about the capacity of networks, Degrees-of-Freedom (DoF) and Generalized Degrees-of-Freedom (GDoF) have been used in recent related researches, frequently. One of the major challenges is how to deal with interference-limited wireless networks where researchers use DoF to analyze it. Interference Alignment (IA), originated in [1], [2], has been introduced to respond demands for higher DoF. Although the traditional methods (like treating interference as noise and TDMA), are satisfying for little networks, they become inefficient for large multi-user networks. For example, a naive scheme, like TDMA, in k-user IC achieves $\frac{1}{k}$ DoF. However, in [2], it has been shown that IA achieves $\frac{1}{2}$ DoF per user and surprisingly, this DoF is independent of k.

In spit of great progresses in presenting IA methods in theory, implementation of them in practice encounters several challenges. In most of IA schemes, instantaneous Channel State Information at Transmitter (CSIT) is strictly required for correct performance of the system. The results in [3], [4] show that, without instantaneous CSIT, DoF of these methods drastically degrades. [5], [6] show that the delayed CSIT can improve DoF compared to the no CSIT case. However, in most of the practical cases, it is not possible to provide even delayed CSIT. Recently, a new method, called Blind Interference Alignment (BIA), has been introduced to align interference without CSIT [7]. In [7], the authors introduce the idea of BIA using the staggered antennas' switching. Relays were also used for BIA in [8], [9] where it is shown that in the case of blind transmitters, the relays having instantaneous or delayed version of CSI can improve DoF. Moreover, using switching multi-mode antennas for BIA have been investigated in [10], [11], where a technique is proposed to change the channel coefficients between transmitters and receivers in a desired way to align interference at the receivers. This technique is based on using the receivers which are equipped with muti-mode antennas. By switching the preset modes at the receivers, these BIA schemes create new independent channels between the transmitters and the receivers. One practical multi-mode antenna system is Electrically Steerable Passive Array Radiator (ESPAR) which is introduced in [12]. Recently, several BIA methods have been presented for some ICs, thanks to multi-mode antennas. In [13], a BIA scheme for the three-user Single-Input Single-Output (SISO) IC has been presented and [14] presents a BIA scheme for the k-user Multiple-Input Single-Output (MISO) IC. Most of researches in this domain focused on *single-hop* ICs and many problems for multi-hop ICs remained unsolved.

Motivated by the wide usage of *multi-hop* ICs, in this paper, we investigate the three-user multi-hop SISO IC. In our network model, the transmitters and relays have conventional antennas. The receivers are equipped with multi-mode antennas and they are capable of switching between potential channels according to antennas' preset modes. Moreover, the transmitters and the relays have no CSI. We present a BIA scheme to manage interference without CSIT. We show that our method achieves maximum achievable sum-DoF for this network. Thus, our BIA scheme is optimal in the sense of DoF. In addition, we investigate a more practical case which fits the scenarios where we can not equip the receivers with multi-mode antennas. In some practical cases, it is not possible to upgrade all potential receivers of a network to

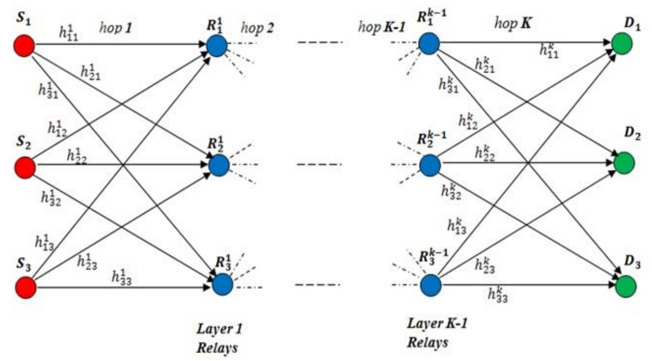

Fig. 1. The three-user multi-hop SISO interference channel.

an advanced system. Also, simplicity of nodes is one of the major advantages for most of networks. In our case, if we want to apply our first proposed BIA scheme to a network, we must equip receivers with multi-mode antennas. As another example, one can consider simple nodes with limited energy resources, like nodes in a Wireless Sensor Network (WSN), as potential receivers. Due to coarse environment of using WSNs, in some applications, we can not access to nodes and when nodes consumed up their resources of energy, we lose them permanently. In this cases, it is reasonable to keep receiver nodes as simple as possible, without losing correct performance of network. Then in this cases, we can not equip the receivers with multi-mode antennas or any sophisticated system. One solution is placing some nodes which are equipped with multi-mode antennas and have suitable conditions in network. These suitable conditions depend on the application and features of network. For example, in mentioned examples, these conditions are access to unlimited resources of energy, accessibility and suitable geographical locations.In order to cope with this challenge, one can augment extra nodes with multimode antennas to the network. In our model, we consider this scenario by assuming that the relays of an intermediate layer have multi-mode antennas instead of the receivers. These relays know global CSI. In this case, we present a BIA method to align interference at the receivers which each one has a single conventional antenna. We show that our method in this case achieves optimal DoF.

The rest of the paper is organized as follows. In Section II, we describe the system model and two considered cases. In Section III, we state our main results for each case and in section IV, we present IA methods for both cases which achieve optimal sum-DoF. In Section V, we conclude the paper.

II. Network Model

In this section, we describe our network model in details. We investigate the three-user k-hop SISO IC. As shown in Fig. 1, this network consists of three transmitters denoted as S_1, S_2 and S_3. The transmitters have conventional antennas. Also, the direct links do not exist between the transmitters and the receivers. The transmitters send their messages to the receivers with the help of the intermediate nodes (i.e., the relays). D_1, D_2 and D_3 denote the receivers which are equipped with multi-mode antennas and have the capability of switching modes of their antennas ,i.e., the corresponding channels between the receivers and the connected nodes to them. There are $k-1$ layers of relays between the transmitters and the receivers. When nodes of a layer send their signals, only nodes in the next layer observe them. Therefor, there are k hops in this IC for arbitrary integer value $k \geq 2$. Each layer of relays consists of three relays; R_1^i, R_2^i and R_3^i denote the first, the second and the third relays at the i^{th} layer, for $i = 1, 2, ..., k-1$. The channel coefficient between the i^{th} node and the j^{th} node in n^{th} hop is denoted as $h_{ji}^n \in \mathbb{C}$ except last hop, for $i, j = 1, 2, 3$ and $n = 1, 2, ..., k-1$. In the last hop, channel coefficients are depends on the modes of the antennas at the receivers. The mode of antenna in D_i at t^{th} channel use is denoted as $l_i(t)$, for $i = 1, 2, 3$. Hence, we use $h_{ji}^k(l_j(t)) \in \mathbb{C}$ for the channel coefficient between the i^{th} relay at the last layer of the relays and D_j at the last hop at t^{th} channel use. In addition, L_i denotes the vector of antenna's mode at D_i. We assume that all channel coefficients in the first $k-1$ hops are constant during all transmissions. The noises at receivers of each hop are additive white Gaussian noises. $Z_{R_i^j}(t) \in \mathbb{C}$ represents the noise in the i^{th} relay at the j^{th} at t^{th} channel use. Also, $Z_{D_i}(l_i(t)) \in \mathbb{C}$ stands for the noise at D_i at t^{th} channel use which depends on $l_i(t)$. All relays are in full-duplex mode, however, they are not instantaneous, as explained in related chapter of [15] in details. All channel coefficients and additive white Gaussian noises are scalars with complex normal distribution with zero mean and unit variance $\mathcal{CN}(0, 1)$ and they are drawn from independent and identical distributions.

In our first considered case, called IC#1 in rest or paper, we assume that the receivers have multi-mode antennas, knowing CSI globally. Since there is not limit on the decoding time, there is no difference between the instantaneous and delayed CSI. In this case, the relays of different hops and the transmitters do not know CSI. Hence, the relays use Amplify-and-Forward (AF) relaying strategy.

Our second model, called IC#2 in the rest of the paper, refers to a more practical scenario. In this case, we assume that the relays at an intermediate layer are equipped with multi-mode antennas. However, the relays in other layers, the transmitters and the receivers are equipped with conventional antennas. Without loss of generality, we assume that the relays at the N^{th} layer are equipped with multi-mode antennas for $1 \leq N \leq k-1$. These relays are capable of switching their modes of antennas and know global CSI instantaneously. However, the receivers are equipped with conventional antennas and know the CSI of the last $N-k$ hops. Other relays and the transmitters do not know CSI of different hops, hence, these relays use AF relaying scheme.

We remark that a multi-mode antenna at the receiver or the transmitter does not create a SIMO or MISO systems, respectively. A node with multi-mode antenna is capable of selecting a mode from preset modes. At any channel use, there is a single active antenna in each mode. For example, in a ESPAR antenna, by applying preset changes in beam pattern of this antenna, mode changes in a desired way. Therefore, each node in our network has a single antenna at each channel use and our both of models are three-user k-hop SISO IC.

Each transmitters wishes to send a message to its corresponding receiver. m_i^j denotes the j^{th} message of S_i for D_i. Also, $u_i^j(t)$ is beamforming coefficient of m_i^j at t^{th} channel use. The value of beam forming coefficient of each message is set to 1 or 0 (i.e., $u_i^j(t) \in \{0, 1\}$). S_i generates $X_{S_i}(t)$ based on its messages and corresponding beamforming coefficients at t^{th} channel use. The relays of each layer observe transmitted signals by the nodes at previous layer. $Y_{R_i^j}(t)$ stands for the received signal at R_i^j and $X_{R_i^j}(t)$ denotes the transmitted signal of R_i^j, at the t^{th} channel use. Moreover, D_i receives $Y_{D_i}(t)$ at t^{th} channel use. Based on its side information, each receiver tries to recover its desired messages from its received signals. In the next section, we investigate optimal DoF of both cases.

III. Main Results

In this section, we investigate the optimal sum-DoF of our two considered ICs, described in the previous section. Two following theorems express maximum achievable sum-DoF of IC#1 and IC#2, respectively. To prove theorems, first, we find an upper bound on the maximum achievable sum-DoF for each case. Then we present two schemes which achieve these upper bounds in the next section.

Theorem 1: For IC#1, maximum achievable sum-DoF equals to $\frac{6}{5}$.

Proof: First, we provide an intuitive proof for this theorem. It proved in [13, Lemma 3] that the achievable sum-DoF using linear scheme for the three-user single-hop SISO IC is upper bounded by $\frac{6}{5}$. In the three-user single-hop SISO IC, each transmitter sends a linear combination of its messages without CSIT. Therefore, each receiver receives a linear combination of all messages of all transmitters. Now assume that there are several hops with relays which use AF strategy without CSI. Again, the transmitters send a linear combination of their messages. Hence, in the first hop, the relays receive combinations of messages all transmitters based on the channel coefficients of the first hop. These relays forward the received signals to the relays at the second layer. Thus, each relay receives a linear combination of all messages. Following similar steps at the next hops, the receivers receive a linear combination of all transmitted signals by the transmitters. Since there is no CIS at the relays, they cannot attempt to facilitate the interference alignment at the receivers. Therefore, each receiver must recover its desired messages from a linear combination of all messages with coefficients based on the channel coefficients of all hops. Thus, situation for receivers is same as single hop case. Therefore, one can see that the achievable sum-DoF for IC#1 is upper bounded by $\frac{6}{5}$, same as the single-hop version. Now, we provide the technical proof of this theorem. The received signals at the receivers at t^{th} channel use can be shown as:

$$\underline{Y}(t) = \underline{H}_k(l_1(t), l_2(t), l_3(t)) \times \underline{H}_{k-1} \times \underline{H}_{k-2} \times \ldots \times \underline{H}_2 \times \underline{H}_1 \times \underline{X}(t). \quad (1)$$

where $\underline{Y}(t) = [Y_{D_1}(t), Y_{D_2}(t), Y_{D_3}(t)]^T$; $\underline{X}(t) = [X_{S_1}(t), X_{S_2}(t), X_{S_3}(t)]^T$; $\underline{H}_i$ denotes the matrix representation of channel coefficients at the i^{th} hop, for $i = 1, 2, ..., k-1$; $\underline{H}_k(l_1(t), l_2(t), l_3(t))$ stands for the channel coefficients of the last hop which (depends on modes of antennas in the receivers). An equivalent transfer matrix $\underline{H}_{eq}(l_1(t), l_2(t), l_3(t))$ is defined considering the multiple hops.

$$\underline{H}_{eq}(l_1(t), l_2(t), l_3(t)) = \underline{H}_k(l_1(t), l_2(t), l_3(t)) \times \underline{H}_{k-1} \times \underline{H}_{k-2} \times \ldots \times \underline{H}_2 \times \underline{H}_1. \quad (2)$$

We assume number of channel uses in our BIA scenario are equal to n and using this scenario D_i achieves d_i DoF. In addition, d_{ij} denotes the number of dimensions of the common subspace projected from D_i and D_j at D_k for $i \neq j, j \neq k, i \neq k$ and $i, j, k \in \{1, 2, 3\}$. Applying [13, Lemma 2] on IC#1 with transfer matrix representation $\underline{H}_{eq}(l_1(t), l_2(t), l_3(t))$ at t^{th} channel use, results in following equations and inequalities over d_i and d_{ij} at different receivers.

$$d_{ij} = d_{ji}; i \neq j, i, j \in \{1, 2, 3\} \quad (3)$$

$$d_{ij} + d_{ik} \leq d_i; i \neq j, i \neq k, k \neq ji, j, k \in \{1, 2, 3\} \quad (4)$$

At D_1, D_2 and D_3, the inequalities in (5), (6) and (7), respectively, show total number of dimensions spanned by received signals. These inequalities are justified by the fact that d_{ij} and d_{ji} overlap at D_k, e.g., $d_{23} = d_{32}$ at D_1.

$$d_1 + d_2 + d_3 - d_{23} \leq n \quad (5)$$

$$d_1 + d_2 + d_3 - d_{13} \leq n \quad (6)$$

$$d_1 + d_2 + d_3 - d_{12} \leq n \quad (7)$$

Combining (5), (6) and (7), we obtain:

$$\frac{d_1 + d_2 + d_3}{n} - \frac{d_{12} + d_{13} + d_{12}}{3n} \leq 1 \quad (8)$$

The second term in the left side of (8) can be bounded using (4), as:

$$\frac{d_{12} + d_{13} + d_{12}}{3n} \leq \frac{d_1 + d_2 + d_3}{6n} \quad (9)$$

Then by substituting (9) in (8), we obtain:

$$\frac{d_1 + d_2 + d_3}{n} \leq \frac{6}{5} \quad (10)$$

Thus, we proved that sum-DoF of IC#1 is upper bounded by $\frac{6}{5}$. In the next section, we present a BIA scheme that achieves this upper bound. This completes the proof.■

Theorem 2: For IC#2, maximum achievable sum-DoF is $\frac{6}{5}$.

Proof: IC#2 can be considered as a concatenation of two ICs. First IC consists of the first N hops. In this IC, the relays of the N^{th} layer, equipping with multi-mode antennas, serve as receivers. Second IC consists of the last $k - N$ hops. In this IC, the relays with multi-mode antennas serve as transmitters

. The receivers and the relays at the N^{th} intermediate layer know CSI of the last $k-N$ hops instantaneously. However, relays at at the rest of layers have no CSI and use AF strategy. Since all operations on signals are linear, by assuming the last $k-N$ hops as an equivalent single-hop channel, we can obtain a single-hop IC in which transmitters and receivers know CSI instantaneously. In [2], it has been shown that in such channels, each receiver can achieve $\frac{1}{2}$ DoF with IA. Hence, for the second IC, optimal sum-DoF is $\frac{3}{2}$. Moreover, it is obvious that the first IC is a version of IC#1 with N hops. Then maximum achievable sum-DoF of the first IC is $\frac{6}{5}$.Thus, the first IC is the bottleneck of IC#2 and the maximum achievable sum-DoF of IC#2 is upper bounded by $\frac{6}{5}$. In the next section, we present a BIA scheme that achieves $\frac{6}{5}$ DoF. This completes the proof.■

IV. BIA Schemes

In this section, we present the BIA schemes for both IC#1 and IC#2 based on switching the multi-mode antennas at the receivers and a layer of relays, respectively.

A. BIA scheme for IC#1

Each transmitter sends two messages to its corresponding receiver over five time slots to achieve sum-DoF of 6/5. m_1^i and m_2^i denote two messages of S_i to D_i for $i=1,2,3$. In each time slot, the transmitters send their signals based on messages and beamforming vectors. u_j^i denotes the beamforming vector of m_j^i.Hence,

$$X_{S_i}(t)=u_1^i(t).m_1^i+u_2^i(t).m_2^i. \tag{11}$$

Each hop contains five time slots in our BIA scheme. We assume that, five time slots of the first hop occur first and then five time slots of the second hop follow. This continues in the same way for the next hops. Eventually, five time slots in k^{th} hop occur and at the end of these time slots, the receivers try to recover their desired messages from its received signals. This procedure does not imply that one round consists of $5\times k$ time slots, since as soon as five time slots of a certain hop are passed, the next round of interference alignment begins at that hop. In addition, since the relays are full-duplex, a relay, like R_i^n, is capable to send $X_{R_i}(t)$ at r^{th} round in the $n+1^{th}$ hop and receives $Y_{R_i^n}(t)$ at $r+1^{th}$ round in the n^{th} hop, instantaneously. Therefore, each round of applying BIA consists of five time slots. In the rest of the paper, we use t_m to denote t^{th} time slot in the m^{th} hop, if needed. We assume that the amplifying gains in all relays are equal to one. For the first hop, at the end of $t_1=t$, we obtain:

$$Y_{R_i}(t)=h_{i1}^1X_{S_1}(t)+h_{i2}^1X_{S_2}(t)+h_{i3}^1X_{S_3}(t)+Z_{R_i}(t) \tag{12}$$

$$\begin{bmatrix} Y_{R_1^1}(t) \\ Y_{R_2^1}(t) \\ Y_{R_3^1}(t) \end{bmatrix} = \begin{bmatrix} h_{11}^1 & h_{12}^1 & h_{13}^1 \\ h_{21}^1 & h_{22}^1 & h_{23}^1 \\ h_{31}^1 & h_{32}^1 & h_{33}^1 \end{bmatrix} \begin{bmatrix} X_{S_1}(t) \\ X_{S_2}(t) \\ X_{S_3}(t) \end{bmatrix}. \tag{13}$$

$$X_{R_i^j}(t_{j+1}=t)=Y_{R_i}^j(t_j=t); i\in\{1,2,3\}, j\in\{1,...,k-1\} \tag{14}$$

As mentioned in Section II, the channel coefficients of the the first $k-1$ hops in IC#1 are constant during all transmissions. However, the channel coefficients of the k^{th} hop may change during transmissions since they dependent on the modes of antennas at the receivers. Substituting (14) in (13), we obtain received signals at the relays of the i^{th} layer, for $i=1,...,k-2$, as:

$$\begin{bmatrix} Y_{R_1^i}(t) \\ Y_{R_2^i}(t) \\ Y_{R_3^i}(t) \end{bmatrix} = \underline{H}_i\times\underline{H}_{i-1}\times\ldots\times\underline{H}_1\times \begin{bmatrix} X_{S_1}(t) \\ X_{S_2}(t) \\ X_{S_3}(t) \end{bmatrix}. \tag{15}$$

At the end of t^{th} time slot in the last hop, the receivers receive signals depending on their modes of antennas at the current time slot:

$$\begin{bmatrix} Y_{D_1}(t) \\ Y_{D_2}(t) \\ Y_{D_3}(t) \end{bmatrix} = \underline{H}_k(l_1(t),l_2(t),l_3(t))\times \begin{bmatrix} X_{R_1^{k-1}}(t) \\ X_{R_2^{k-1}}(t) \\ X_{R_3^{k-1}}(t) \end{bmatrix}. \tag{16}$$

where

$$\underline{H}_k(l_1(t),l_2(t),l_3(t)) = \begin{bmatrix} h_{11}^k(l_1(t)) & h_{12}^k(l_1(t)) & h_{13}^k(l_1(t)) \\ h_{21}^k(l_2(t)) & h_{22}^k(l_2(t)) & h_{23}^k(l_2(t)) \\ h_{31}^k(l_3(t)) & h_{32}^k(l_3(t)) & h_{33}^k(l_3(t)) \end{bmatrix} \tag{17}$$

Thus, we have:

$$\begin{bmatrix} Y_{D_1}(t) \\ Y_{D_2}(t) \\ Y_{D_3}(t) \end{bmatrix} = \mathbb{A}\times \begin{bmatrix} u_1^1(t)m_1^1+u_2^1(t)m_2^1 \\ u_1^1(t)m_1^1+u_2^1(t)m_2^1 \\ u_1^1(t)m_1^1+u_2^1(t)m_2^1 \end{bmatrix}. \tag{18}$$

$$\mathbb{A} = \begin{bmatrix} a_{11}(l_1(t)) & a_{12}(l_1(t)) & a_{13}(l_1(t)) \\ a_{21}(l_2(t)) & a_{22}(l_2(t)) & a_{23}(l_2(t)) \\ a_{31}(l_3(t)) & a_{32}(l_3(t)) & a_{33}(l_3(t)) \end{bmatrix}.$$

where $\mathbb{A}=\underline{H}_{eq}(l_1(t),l_2(t),l_3(t))$ is used it for brevity. In order to gain insights of the interference alignment scheme at the receivers, we investigate coefficient of messages during five time slots at each receiver. First, we consider the received signals at D_1 and then we present sufficient conditions on beamforming vectors and modes of antennas to ensure the alignment of the spaces due to two messages at a receiver. We denote the vector of received signal at D_i during five time slots by following equation for $i=1,2,3$:

$$\mathbb{Y}_i=\mathbb{R}_i\times\underline{S}. \tag{19}$$

$$\mathbb{Y}_i=\begin{bmatrix} Y_{D_i}(1), & Y_{D_i}(2), & \ldots, & Y_{D_i}(5) \end{bmatrix}^T;$$
$$\underline{S}=\begin{bmatrix} m_1^1, & m_2^1, & m_1^2, & m_2^2, & m_1^3, & m_2^3, \end{bmatrix}^T.$$

The column in R_i which is related to m_k^j is denoted by:

$$v_j^k(i)=\begin{bmatrix} a_{ij}(l_j(1))u_k^j(1) \\ a_{ij}(l_j(2))u_k^j(2) \\ a_{ij}(l_j(3))u_k^j(3) \\ a_{ij}(l_j(4))u_k^j(4) \\ a_{ij}(l_j(5))u_k^j(5) \end{bmatrix};\quad \begin{matrix} i,j\in\{1,2,3\} \\ k\in\{1,2\} \end{matrix} \tag{20}$$

Hence, for aligning two vectors, like $v_k^j(i)$ and $v_{k'}^{j'}(i)$, it is sufficient to apply the following conditions to the related beamforming vectors and modes of antenna at D_i.

$$d(\underline{u}_k^j, \underline{u}_{k'}^{j'}) = 0. \quad (21)$$

$$\forall t \neq t' \quad if \quad u_k^j(t).u_{k'}^{j'}(t') = 1 \Rightarrow l_i(t) = l_i(t') \quad (22)$$

where $i, j, j' \in \{1, 2, 3\}$, $k, k' \in \{1, 2\}$ and $t, t' \in \{1, 2, \dots, 6\}$. Also, $d(\underline{u}_k^j, \underline{u}_{k'}^{j'})$ shows the hamming distance between two vectors $\underline{u}_k^j$ and $\underline{u}_{k'}^{j'}$. The above conditions explain that to align two messages at a certain receiver in a common dimension: (i) we must use the same beamforming vectors for them, (ii) when they are involved in the signals of the related transmitters, the modes of antenna in the intended receiver must set to a predefined mode.

With mentioned conditions, one can find many (i.e., non-unique) beamforming vectors and vectors of modes of antenna. In order to achieve $\frac{6}{5}$ DoF, our scheme must align two undesired messages at each receiver. Thus, we pick up 2 undesired messages for each receiver and find the proper beamforming vectors for these messages and antenna's mode vector. We assume that the multi-mode antennas at the receivers are similar with two available mode named 1 and 2. The following vectors satisfy the required conditions in (21) and (22).

$$\underline{u}_1^2 = \underline{u}_1^2 = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \end{bmatrix}; L_3 = \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \end{bmatrix}$$
$$\underline{u}_2^2 = \underline{u}_1^3 = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \end{bmatrix}; L_1 = \begin{bmatrix} 1 & 2 & 2 & 1 & 2 \end{bmatrix}$$
$$\underline{u}_2^3 = \underline{u}_1^1 = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \end{bmatrix}; L_2 = \begin{bmatrix} 2 & 1 & 1 & 1 & 2 \end{bmatrix}$$

Now we demonstrate the alignment of two unintended messages on a single dimension at each receivers. Applying the above vectors to the transmitters and the receivers' antenna results in:

$$\mathbb{R}_1 = \begin{bmatrix} 0 & a_{11}(1) & a_{12}(1) & 0 & 0 & 0 \\ a_{11}(2) & 0 & 0 & a_{12}(2) & a_{13}(2) & a_{13}(2) \\ a_{11}(2) & a_{11}(2) & a_{12}(2) & a_{12}(2) & a_{13}(2) & a_{13}(2) \\ a_{11}(1) & a_{11}(1) & a_{12}(1) & 0 & 0 & a_{13}(1) \\ 0 & 0 & 0 & a_{12}(2) & a_{13}(2) & 0 \end{bmatrix} \quad (23)$$

Hence, at the first receiver, m_2^2 and m_1^3 occupy the same dimension according to 4^{th} and 5^{th} columns of $\mathbb{R}_1$. Also, at the second and third receivers, messages occupy dimensions as follow:

$$\mathbb{R}_2 = \begin{bmatrix} 0 & a_{21}(2) & a_{22}(2) & 0 & 0 & 0 \\ a_{21}(1) & 0 & 0 & a_{22}(1) & a_{23}(1) & a_{23}(1) \\ a_{21}(1) & a_{21}(1) & a_{22}(1) & a_{22}(1) & a_{23}(1) & a_{23}(1) \\ a_{21}(1) & a_{21}(1) & a_{22}(1) & 0 & 0 & a_{23}(1) \\ 0 & 0 & 0 & a_{22}(2) & a_{23}(2) & 0 \end{bmatrix} \quad (24)$$

$$\mathbb{R}_3 = \begin{bmatrix} 0 & a_{31}(1) & a_{32}(1) & 0 & 0 & 0 \\ a_{31}(2) & 0 & 0 & a_{32}(2) & a_{33}(2) & a_{33}(2) \\ a_{31}(1) & a_{31}(1) & a_{32}(1) & a_{32}(1) & a_{33}(1) & a_{33}(1) \\ a_{31}(1) & a_{31}(1) & a_{32}(1) & 0 & 0 & a_{33}(1) \\ 0 & 0 & 0 & a_{32}(2) & a_{33}(2) & 0 \end{bmatrix} \quad (25)$$

Thus, at the second receiver, m_1^1 and m_2^3 locate in one dimension according to the first and last columns of $\mathbb{R}_2$. Moreover, at D_3, m_2^1 and m_1^2 occupy one dimension according to the second and third columns of $\mathbb{R}_3$. Therefore, at all receivers, two desired messages occupy two dimensions and undesired messages (i.e., interference messages) occupy three remained dimensions. Therefore, the receivers are cable of recovering both of their desired messages successfully. At the end of BIA scheme, each receiver receives its two messages during five time slots and achieves $\frac{2}{5}$ DoF. Since our scheme achieves $\frac{6}{5}$ DoF which is the upper bound of maximum achievable sum-DoF.

We remark that there is no limit on the number of preset modes at the receivers' antennas and we can use antennas with different number of preset modes at the receivers. For example, with following vectors, we can also implement BIA scheme successfully with same beamforming vectors which we used in this section.

$$L_1 = \begin{bmatrix} 3 & 2 & 2 & 1 & 2 \end{bmatrix}.$$
$$L_2 = \begin{bmatrix} 4 & 3 & 3 & 3 & 1 \end{bmatrix}.$$
$$L_3 = \begin{bmatrix} 2 & 1 & 2 & 2 & 3 \end{bmatrix}.$$

B. BIA scheme for IC#2

Now, we sate the BIA scheme for IC#2. As described in section III, we can assume this network as concatenation of two networks. Both networks are three-user multi-hop SISO ICs. The first network is same as IC#1. At the second network, the relays which are equipped with multi-mode antennas, serve as transmitters and they know CSI globally. Also, the receivers know CSI of second network. Since the relays between the transmitters and the receiver do not have CSI and use AF strategy, this network is a three-user SISO single hop channel. The k-user interference channel with instantaneous CSI at the transmitters and receivers was investigated in [2]. By using presented method in [2], each receiver achieves $\frac{1}{2}$ DoF. Therefore, the sum-DoF is $\frac{k}{2}$ for the k-user IC and $\frac{3}{2}$ for our case. Thus, presented IA method in [2] can transfer six messages from the transmitters to receivers over four time slots. Our presented scheme consists of two phase based on two parts of IC#2.

Phase#1- This phase consists of five time slots in each hop. In five time slots, we apply our presented BIA scheme for IC#1 on the first N hops of the network. Scheduling of time slots occurrence in these hops is same as before. At the end of this phase, the relays at the N^{th} layer of relays can recover messages of their corresponding receivers successfully. Thus, at the end of phase#1, R_i^N knows m_1^i and m_2^i, for $i = 1, 2, 3$.

Pahse#2- This phase consists of four time slots and we apply presented IA scheme in [2] on the the last $k - N$ hops. This IA method is feasible here, because the receivers know CSI of the last $k - N$ hops. Since relays at the N^{th} intermediate layer use multi-mode antennas for transmitting and receiving signals, we can not define new modes for these antennas during phase#2. During Phase#2 at the second part of IC#2, these relays receive the signals of phase#1 of the next IA round.

Thus, antenna modes during four time slots of phase#2 are same as the last fourcorresponding modes in phase#1. At the end of this phase, the receivers recover their desired messages. Then this method achieves $\frac{2}{5}$ DoF for each receiver and $\frac{6}{5}$ in total as sum-DoF.

V. Conclusion

In this paper, we investigated the three-user SISO multi-hop interference channel. We considered a scenario where each receiver is equipped with a multi-mode antenna and has CSI of all hops. However, the relays and the transmitters have conventional antennas and do not know CSI. We proved that maximum achievable sum-DoF of this network is upper bounded by $\frac{6}{5}$. Then we presented a BIA scheme which achieved this upper bound based on switching modes of antennas at the receivers. Also, we derived two conditions for beamforming vectors of messages and modes of antennas for successful alignment of two messages. These conditions imply that if we want to align two messages at a certain receiver, the hamming distance between beamforming vectors of two messages must be equal to zero and where-ever the beamforming coefficient of these messages is 1, the antenna mode must adjust to a certain mode. With this conditions, we can conclude the set of values for beamforming vectors and modes of antennas vectors which results successful BIA, are not unique. Also, we showed that one can use different antennas with different number of preset modes in each receiver. Then we investigated a scenario where it is preferable to equip some relays with multi-mode antennas instead of the receivers. We considered this network as a concatenation of two sub-networks and derived the maximum achievable sum-DoF as $\frac{6}{5}$. We also presented a BIA scheme for this case.

References

[1] M. Maddah-Ali, A. Motahari, and A. Khandani, "Communication over mimo X channels: Interference alignment, decomposition, and performance analysis," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3457-3470, 2008.

[2] V. R. Cadambe and S. A. Jafar, "Interference Alignment and Degrees of Freedom of the K-User Interference Channel," *IEEE Trans. Inf. Theory*, vol.54, pp. 3425-3441, 2008.

[3] Y. Zhu and D. Guo, "The Degrees of Freedom of Isotropic MIMO Interference Channels without State Information at Transmitters", *IEEE Trans. Inf. Theory*, vol. 58, pp. 341-352, Jan. 2012.

[4] C. Huang, S. A. Jafar, S. Shamai and S. Vishwanath, "On Degrees of Freedom Region of MIMO Networks Without Channel State Information at Transmitters", *IEEE Trans. Inf. Theory*, vol. 58, pp. 849-857, Feb. 2012.

[5] M. A. Maddah-Ali and D. Tse, "Completely stale transmitter channel state information is still very useful," Oct. 2010, Available: http://arxiv.org/abs/1010.1499.

[6] C. S. Vaze, M. K. Varanasi, "Degrees of freedom region for the two-user MIMO broadcast channel with delayed CSI," in to be presented, *IEEE Trans. symp. Inf. Theory*, St. Petersburg, Russia, Aug. 2011, Available: http://arxiv.org/abs/1101.0306.

[7] C. Wang, T. Gou, and S. A. Jafar, "Aiming perfectly in the darkblind interference alignment through staggered antenna switching," *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2734-2744, Jun. 2011.

[8] Y. Tian and A. Yener, "Guiding blind transmitters: Degrees of freedomoptimal interference alignment using relays," *IEEETrans. Inf. Theory*, vol. 59, no. 8, pp.4819 -4832 2013.

[9] D. Frank, K. Ochs, A sezgin, "A Systematic Approach for Interference Alignment in CSIT-less Relay-Aided X-Networks" Sep. 2013 Available: http://www.arxiv.org/abs/1309.3446v1.

[10] R. Qian, M. Sellathurai, D. Wilcox "On the design of Blind Interference Alignment using ESPAR Antenna" *7th International ICST Conference on Communications and Networking*, pp. 866-870. Aug. 2012.

[11] R. Qian, M. Sellathurai "Performance of the Blind Interference Alignment using ESPAR Antennas" *IEEE International Conference on Communications (ICC)*, pp. 4885-4889. Jun. 2013.

[12] K. Gyoda, T. Ohira, "Design of electrically steerable passive array radiator (ESPAR) antennas," *Proc. IEEE Antennas and Propagation Society Intern. Symp.*, vol. 2, pp. 922-925, 2004.

[13] C. Wang, "Degrees of Freedom Characterization: The 3-User SISO Interference Channel with Blind Interference Alignment," *IEEE Communications letters*, vol. 18, no. 5, pp. 757-600. May. 2014.

[14] Y. Lu, W. Zhang, "Blind Interference Alignment in the k-User MISO Interference Channel," *Signal Processing for Communications Symposium Globecom*, pp. 3464-3469. Dec. 2013.

[15] A. El Gamal, Y. H. Kim, *Network Information Theory*, 2011 :CambridgeUniv. Press